Exercise 26.91

The disk has a uniform positive surface charge density \( \sigma \) on its surface.

Since charge is uniformly distributed, the infinitesimal ratio \( \frac{dQ}{dA} = \sigma \)

\[ dQ = \sigma \, dA \]

To simplify the problem, set the integral in Polar form.

Area differential, \( dA = r \, dr \, d\theta \)

\[ Q = \iint_{C_1} \sigma \, r \, dr \, d\theta \]

The region \( C_1: R_1 \leq r \leq R_2, 0 \leq \theta \leq 2\pi \)

\[ Q = \int_{\theta=0}^{\theta=2\pi} \int_{r=R_1}^{r=R_2} \sigma \, r \, dr \, d\theta = \sigma \int_{r=R_1}^{r=R_2} \left[ \frac{r^2}{2} \right] d\theta \]

\[ Q = \frac{\sigma}{2} \left( R_2^2 - R_1^2 \right) \int_{\theta=0}^{\theta=2\pi} d\theta = \frac{\sigma}{2} \left( R_2^2 - R_1^2 \right) (2\pi) \]

\[ Q = \sigma \pi \left( R_2^2 - R_1^2 \right) \]
Exercise 21.91

For an arbitrary Point Point on the x-axis, find the magnitude of the Electric Field $E$.

Due to symmetry, the components of $E_x, E_y, E_z$ will cancel each other for any point between $R_1$ and $R_2$.

Only the component along the x-axis is considered. Let $r$ be any point between $R_1$ and $R_2$, $R_1 \leq r \leq R_2$.

$dE_x = dE \sin \phi = \frac{dQ}{(x^2 + r^2)^{3/2}} = \frac{\sigma Kx}{(x^2 + r^2)^{3/2}}$

$\theta = \frac{x}{x^2 + r^2}$

$dQ = \sigma r dr d\theta$

$dE_x = \frac{\sigma Kx r dr d\theta}{(x^2 + r^2)^{3/2}}$

$E_x = \int_{r = R_1}^{r = R_2} \int_{\theta = 0}^{\theta = 2\pi} \frac{\sigma Kx r dr d\theta}{(x^2 + r^2)^{3/2}}$

$E_x = 2\pi - \sigma Kx \left( \frac{1}{\sqrt{x^2 + R_2^2}} - \frac{1}{\sqrt{x^2 + R_1^2}} \right)$

$E_x = 2\pi - \sigma Kx \left( \frac{1}{\sqrt{x^2 + R_2^2}} - \frac{1}{\sqrt{x^2 + R_1^2}} \right)$

$\omega = \frac{1}{2\pi} \sqrt{\frac{\sigma}{2\varepsilon_0 m}} \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$